THE ANALYSIS OF GROUPS OF EXPERIMENTS INVOLVING SEVERAL FACTORS

BY

B. N. TYAGI, O. P. KATHURIA AND P. P. RAO

Institute of Agricultural Research Statistics, New Delhi

(Received in June, 1970)

1. INTRODUCTION

In agricultural field experimentation the results of an experiment conducted at a particular place or in a particular year are not of much practical use unless the experiment is repeated at a number of places over a number of years. The results of the experiment after pooling over a number of places or years will be more broad-based and more stable and help the research workers in formulating their future experimental programmes and the extension workers in disseminating information for practical farming. The statistical problems involved in combining the results of similar experiments conducted over space or time have been extensively dealt with by Yates and Cochran (3), Cochran (2), Cochran and Cox (1) and others. These authors, however, dealt with experiments involving one factor only. In the case of experiments involving several factors, one is often interested in studying the behaviour of interactions of various factors with different years or different places. The differential behaviour of various effects in different years or places will affect the conclusions that may be drawn from the set of experiments under study.

The object of this paper is to present methods for combining results of similar factorial experiments conducted over a number of years or at a number of places particularly in cases when the data are available in the form of two-way tables of means along with their standard errors. In such cases no information is available about second and higher order interactions between factors. Therefore, these interactions have been ignored from the combined analysis without any loss of information.

2. Combination of Results of a Single Factor Experiment

Consider an experiment with t treatments laid out in r randomised blocks conducted for p years. The usual linear model would be

$$y_{ijk}=\mu+p_i+r_{ij}+t_k+(pt)_{ik}+(tr)_{ijk}$$

where p_i ($i=1, 2,.. p$) is the effect of the ith year, r_{ij} ($j=1, 2,...r$) is the effect of the jth replicate in the ith year, t_k ($k=1, 2,...t$) is the effect of kth treatment, $(pt)_{ik}$ is the interaction of kth treatment with ith place and $(tr)_{ijk}$ is the random error. The parameters μ, r_{ij} and t_k are constants, and others are random variables with

$$E(p_i)=E(pt)_{ik}=E(tr)_{ijk}=0,$$
$$E(p_i)^2=\sigma_p^2,$$
$$E(tr)^2_{ijk}=\sigma_i^2;$$

We divide the treatment×places interaction into $(t-1)$ orthogonal contrasts each carrying $(p-1)$ $d.f.$; and the corresponding random variable being denoted by $(pt)_l$ $(l=1, 2,...\overline{t-1})$ with variance $\sigma^2_{(pt)l}$.

This leads to the following analysis of variance:

TABLE 1

Analysis of variance of experiment with single factor treatments

Source of variation	*d. f.*	*M.S.S.*	*Expected value of M.S.S.*
Years	$(p-1)$	—	—
Replications	$p(r-1)$	—	—
Treatments	$(t-1)$	T	$\frac{1}{p}\sum_{i=1}^{p}\sigma_i^2+\frac{r}{(t-1)}\sum_{l=1}^{t-1}\sigma^2_{(pt)l}$ $+\frac{rp}{t-1}\sum_{k}(t_k-\bar{t})^2$
Treatments × Years	$(p-1)(t-1)$	TP	$\frac{1}{p}\sum_{i=1}^{p}\sigma_i^2+\frac{r}{(t-1)}\sum_{l=1}^{t-1}\sigma^2_{(pt)l}$
Error	$p(r-1)(t-1)$	E	$\frac{1}{p}\sum_{l=1}^{p}\sigma_i^2$
Total	$tpr-1$		

From the above table the estimates of valid errors for differences between two treatment means averaged over all the years can be obtained. But the estimates of error for mean differences between two treatments over a specified set of years cannot be obtained from the mean sum of the squares if $\sigma_i^2 (i=1, 2, \ldots p)$ are not equal and in such a case ordinary tests of significance would not hold. However, if we assume that $\sigma_i^2 = \sigma^2$ for all $i=1, 2, \ldots p$, the expected values of various mean sums of squares are given by

$$E(T) = \sigma^2 + \frac{r}{t-1} \sum_{l=1}^{t-1} \sigma^2_{(pt)l} + \frac{rp}{(t-1)} \sum_{k=1}^{t} (t_k - \bar{t})^2$$

$$E(TP) = \sigma^2 + \frac{r}{t-1} \sum_{l=1}^{t-1} \sigma^2_{(pt)l}$$

and

$$E(E) = \sigma^2$$

If, further, it is assumed that $\sigma^2_{(pt)l} = \sigma^2_{pt}$ for all treatments we have

$$E(T) = \sigma^2 + r\sigma^2_{pt} + \frac{rp}{t-1} \sum_{k=1}^{t} (t_k - \bar{t})^2$$

$$E(TP) = \sigma^2 + r\sigma^2_{pt}$$

and

$$E(E) = \sigma^2$$

To sum up we conclude that

(*i*) If $\sigma^2_i = \sigma^2$ and $\sigma^2_{(pt)l} = \sigma^2_{pt}$ for all treatment comparisons, and the observations are assumed to be normal, *TP* can be compared with *E* by the *F*-test. And, if *TP* is significant, *T* may be compared with *TP* by the *F*-test. But if *TP* is not significant, *T* is to be compared with pooled estimate of error obtained by pooling *TP* with *E*. However, if $\sigma^2_{(pt)l}$'s are not all equal, we have to divide the sums of squares relating to treatments and interactions into separate components and then separately compare each component of treatment sums of squares against the corresponding component of treatment × year interaction.

(*ii*) If σ_i^2 are not homogeneous as shown by the Bartlett's test, then *TP* and *T* of Table 1 cannot be compared with *E*. Weighted analysis of variance has to be carried out, the weights being the inverse of the per unit variance in the individual years. If the weighted analysis indicates the significance of *TP*, it can be concluded that the interaction part *i.e.* $\sigma^2_{(pt)l}$ in the mean sum of squares for the interaction *TP* is more dominant and the error part can be considered negligible. Under these assumptions

$$E(T) = \frac{r}{t-1} \sum_{l=1}^{t-1} \sigma^2_{(pt)l} + \frac{rp}{t-1} \sum_{k=1}^{t} (t_k - \bar{t})^2$$

and $$E(TP)=\frac{r}{(t-1)}\sum_{l=1}^{t-1}\sigma^2_{(pt)l}\,.$$

Thus, the heterogeneity of error variances does not influence the estimates of error for the mean differences of treatments. However, if the weighted analysis does not indicate the significance of *TP* there is no satisfactory way either for obaining estimates of error variance or for tests of significance.

3. Combination of Results of Experients Involving Several Factors :

3.1. *Error variances are homogeneous :*

Let us consider an experiment with two factors *A* and *B* with levels *a* and *b* respectively conducted over a period of *p* seasons. Let *r* be the number of replications of the experiment in each season. We assume that the seasons under study provide a representative sample of the entire population of seasons in the experimental area. The linear model would be as under.

$$y_{ijk}=\mu+p_i+\alpha_j+\beta_k+(\alpha\beta)_{jk}+(p\alpha)_{ij}+(p\beta)_{ik}+(p\alpha\beta)_{ijk}+\bar{e}_{ijk}.$$
$$(i=1,2,\ldots\ldots p,\ j=1,\ 2,\ldots\ldots a,\ k=1,\ 2,\ \ldots\ldots b)$$

where α_j, β_k and $(\alpha\beta)_{jk}$ represent the effects of factors *A*, *B* and the interaction *AB*, $(p\alpha)_{ij}$, $(p\beta)_{ik}$ and $(p\alpha\beta)_{ijk}$ represent the effect of interaction of factors *A*, *B* and *AB* with years and $\bar{e}_{ijk}$ is the experimental error averaged over *r* replications of each experiment.

Further, we have

$$\sum_j\alpha_j=\sum_k\beta_k=0,\ \sum_j(\alpha\beta)_{jk}=\sum_k(\alpha\beta)_{jk}=0$$

$$E(p_i)=0,\qquad E(p_i^2)=\sigma_p^2$$

$$E(p\alpha)_{ij}=0,\ E[(p\alpha)_{ij}^2]=\sigma^2_{p\alpha}$$

$$E(p\beta)_{ik}=0,\ E[(p\beta)_{ik}^2]=\sigma^2_{p\beta}$$

$$E(p\alpha\beta)_{ijk}=0,\ E[(p\alpha\beta)_{ijk}^2]=\sigma^2_{p\alpha\beta}$$

$$E(\bar{e}_{ijk})=0,\ \mathrm{E}(\bar{e}^2_{ijk})=\sigma^2$$

Under this model, the analysis of variance of the experiment may be written as in Table 2 below :

TABLE 2

Analysis of variance for an experiment with two factors

Variation	*d. f.*	*M.S.S.*	*Expected value of M.S.S.*
Year	$(p-1)$		–
A	$(a-1)$	S_A	$\sigma_e^2+br\sigma_{p\alpha}^2+\frac{prb}{(a-1)}\Sigma\alpha_j^2$
B	$(b-1)$	S_B	$\sigma_e^2+ar\sigma_{p\beta}^2+\frac{pra}{(b-1)}\Sigma\beta_k^2$
$A\times B$	$(a-1)(b-1)$	S_{AB}	$\sigma_e^2+r\sigma_{p\alpha\beta}^2+\frac{pr}{(a-1)(b-1)}\Sigma(\alpha\beta)_{jk}^2$
Years$\times A$	$(p-1)(a-1)$	S_{PA}	$\sigma_e^2+br\sigma_{p\alpha}^2$
Years$\times B$	$(p-1)(b-1)$	S_{PB}	$\sigma_e^2+ar\sigma_{p\beta}^2$
Years$\times A\times B$	$(p-1)(a-1)(b-1)$	S_{PAB}	$\sigma_e^2+r\sigma_{p\alpha\beta}^2$
Pooled error	$p(r-1)(ab-1)$	E	σ_e^2

It is obvious from Table 2 that S_{PA} , S_{PB}, S_{PAB} can be compared with E by the usual F-test, and in accordance with results of section 2, if these interactions are significant, these can be used for comparing S_A , S_B and S_{AB}. In case, some (or none) of S_{PA}, S_{PB} and S_{PAB} are (is) significant, then only such interaction mean squares (*i.e.*, S_{PA}, S_{PB} or S_{PAB} as the case may be) can be used for testing the significance of the corresponding factorial effects. The other factorial effects would be tested against the pooled estimates of error. However, in factorial experiments with only a few number of levels of A and B and repeated only for a small number of years, say 3 or 4, the degrees of freedom associated with S_{PA}, S_{PB} and S_{PAB} are usually inadequate to provide a reliable estimate of the interaction variance. Therefore, if S_{PA}, S_{PB} and S_{PAB} are homogeneous, these should be pooled and the pooled mean sum of squares should be used for testing S_A , S_B and S_{AB}. The pooled interaction mean sum of squares would then have sufficient degrees of freedom.

In the case of more than two factors, say m factors at $s_1, s_2, \ldots\ldots,$ s_m levels with results given in the form of two-way tables of means along with standard errors for each year, we can work out the mean sums of squares for main effects, first order interaction and their interaction with years, say $SA1$, $SA2$, ... SAm, $SA1A2$, ... SA_{m-1}, A_m and S_{PA2},......, $S_{PA_{m-1}}Am$. But we cannot work out the mean sums of squares for higher order interactions. The incomplete table of analysis of variance of such type of experiments would be as given in Table 3.

TABLE 3

Analysis of variance of experiment with several factors

Sources	d. f.	M. S.S.
Years	$(p-1)$	—
Factorial effects		
A_1	(s_{1-1})	S_{A_1}
A_2	(s_2-1)	S_{A_2}
....	....	....
....	....	....
....	....	....
A_m	(s_m-1)	S_{A_m}
A_1A_2	$(s_1-1)(s_2-1)$	$S_{A_1A_2}$
....	....	....
....	....	....
....	....	....
$A_{m-1}A_m$	$(s_{m-1}-1)(s_m-1)$	$S_{A_{m-1}A^m}$
Interactions (with years)		
S_{PA_1}	$(p-1((s_1-1)$	S_{PA_1}
S_{PA_2}	$(p-1)(s_2-1)$	S_{PA_2}
....	....	....
....	....	....
....	....	....
S_{PA_m}	$(p-1)(s_m-1)$	S_{PA_m}
$S_{PA_1A_2}$	$(p-1)(s_1-1)(s_2-1)$	$S_{PA_1A_2}$
$S_{PA_1A_3}$	$(p-1)(s_1-1)(s_3-1)$	$S_{PA_1A_3}$
....	....	....
....	....	....
....	....	....
$S_pA_{m-1}A_m$	$(p-1)(s_{m-1}-1)(s_m-1)$	$S_{PA_{m-1}}A_m$
Error	$p(r-1)(s_1s_2...s_m-1)$	E

The interaction mean sums squares S_{PA_i} and $S_{PA_iA_j}$ $(i \neq j = 1, 2 ... m)$ should first be tested against the error mean square E for knowing their presence or otherwise. Those of the interaction sums of squares which are present will be pooled if they are found to be homogeneous and test the corresponding factorial effects against this pooled mean sum of squares. These interactions which are not present may be pooled with E and the corresponding effects can be compared with the pooled E. If the interaction mean sums of squares S_{PA_i} and $S_{PA_iA_j}$ which are present $(i \neq j = 1, 2 ... m)$ are not homogeneous, we can divide them into groups such that there is homogeneity within the groups. These groupwise mean sums of squares of interactions can be used for comparing the corresponding factorial effects.

3.2. *Error variances are heterogeneous*

As in the case of single factor experiments the analysis of factorial experiments becomes quite complicated when error variances are heterogeneous. The basic principles of statistical analysis are those given in section 2; the working of various mean sums of squares for a factorial experiment is not so straightforward and is therefore discussed below.

Let us take an experiment with the three factors as A, B, C at levels a, b, c respectively conducted for p seasons. We shall assume that the number of replications in each experiment is constant $(=r)$. Bartlett's test of homogeneity of error variances shows that $S_i^2 (i = 1, 2 ... p)$ are heterogeneous. With three factors we have three two-way tables of means. Consider first the $A \times B$ table of means of all years. Write the means of $(a \times b)$ treatment combinations obtained in each year in the form of a two-way table. Call it $(AB) \times$ Year table of means. Let x_{ijk} be the mean of the j-kth treatment combination of factors A and B in the ith year, averaged over the levels of C. The various steps for finding out the *m.s.s.* and testing the interactions $A \times$ Years, $B \times$ Years and $A \times B \times$ Years, etc. are given in table 4.

TABLE 4

Treatment combinations of A and B	*Years* 1	2	...	p	$T_{jk}=\sum\limits_{i=1} w_i x_{ijk}$
A_1B_1	x_{111}	x_{211}	...	x_{p11}	T_{11}
A_1B_2	x_{112}	x_{212}	...	x_{p12}	T_{12}
....	....	....	...	....	....
....	....	....	...	....	....
A_1B_b	x_{11b}	x_{21b}	...	x_{p1b}	T_{1b}
A_2B_1	x_{121}	x_{221}	...	x_{p21}	T_{21}
A_2B_2	x_{122}	x_{222}	...	x_{p22}	T_{22}
....	....	....	...	....	....
....	....	....	...	....	....
....	....	....	...	....	....
A_aB_b	x_{1ab}	x_{2ab}	...	x_{pab}	T_{ab}
$\sum\limits_{j,k=1}^{a,b} x_{ijk}=P_i$	P_1	P_2............ ..P_p			
$w_i=\dfrac{r.c}{S_i^2}$	w_1	w_2..............w_p			$\sum\limits_{i=1}^{p} w_i=W$
w_iP_i	w_1P_1	w_2P_2.......... w_pP_p			$\sum\limits_{i=1}^{p} w_iP_i=G=\sum\limits_{j,k=1}^{a,b} T_{jk}$
$\sum\limits_{j,k=1}^{a,b} x_i{}^2{}_{jk}=X_i$	X_1	X_2..............X_p			$C=\dfrac{G^2}{a\times b\times W}$

$$\text{Total S.S.}=\sum_{i=1}^{p} w_iX_i-C \qquad \ldots(1)$$

$$\text{Year S.S.}=\sum_{i=1}^{p} w_iP_i^2-C \qquad \ldots(2)$$

$$(AB)\ \text{S.S.}=\frac{1}{W}\sum_{j,k=1}^{a,b} T^2{}_{jk}-C \qquad \ldots(3)$$

Therefore, (AB) × Years Interaction S.S.=(1)−(2)−(3).

Now consider the table of means of factor $A\times$Years. Let $x_{ij.}$ be the mean of the jth level of A in the ith year, averaged over levels of factors B and C. Various steps for finding out the sums of squares for A and $A\times$Years are given in Table 5.

TABLE 5

Treatment	Years 1	2	i	p	$\sum_{i=1} w_i x_{ij.} = T_{j.}$
A_1	$x_{11.}$	$x_{21.}$		$x_{p1.}$	$T_{1.}$
....	....	....	....	....	....
....	....	....	....	....	....
A_j	....	....	$x_{ij.}$	....	$T_{j.}$
....	....	....	....	....	....
....	....	....	....	....	....
A_a	$x_{1a.}$	$x_{ia.}$		$x_{pa.}$	$T_{a.}$
$\sum_{j=1}^{a} x_{ij.} = P_i'$	P_1'	P_2'	P_i'	P_p'	
$\frac{r.b\,c.}{S_i^2} = w_i'$	w_1'	w_2'	w_i'	w_p'	$\sum_{i=1}^{p} w_i' = W'$
$\sum_{j=1}^{a} x^2_{ij.} = X_1'$	X_1'	X_2'	X_i'	X_p'	
$w_i'\,P_i'$	$w_1'\,P_1'$	$w_2'\,P_2'$	$w_i'P_i'$	$w_p'\,P_p'$	$\sum_{i=1}^{p} w_i'P_i' = G' = \sum_{j=1}^{a} T_j$

Total S.S. $= \sum_{i=1}^{p} w_i' X_i' - C'$ where $C' = \frac{G'^2}{aW'}$

S.S. between Years $= \frac{1}{p} \sum_{i=1}^{p} w_1'\, P_i'^2 - C'$

S.S. due to factor A $= \frac{1}{W'} \sum_{j=1}^{a} T_j^2. - C'$

$(A\times Y)$ interaction S.S. may be obtained by subtraction, In a similar manner the sums of squares for B and $B\times$Years interaction can be obtained by working out suitable weights in the $B\times$Years table of means. $(A\times B\times Y)$ Interaction S.S. is obtained by sub-traction *i.e.*

$$(A\times B\times Y)\text{ S.S.}=(AB)\times\text{Years S.S.}-(A\times Y)\text{ S.S.}-(B\times Y)\text{ S.S.}$$

In order to test for the presence of the interactions $A\times Y$, $B\times Y$ and $A\times B\times Y$ we follow the procedure as given by Cochran and Cox (1). As they are, the sums of squares due to $A\times Y$, $B\times Y$ and $A\times B\times Y$ are not distributed as χ^2. Therefore they are reduced to quantities that are distributed approximately as χ^2. We thus get the quantities,

$$\frac{(n-4)(n-2)}{n(n+a-3)}\times\left[(A\times Y)\text{S.S.}\right],\ \frac{(n-4)(n-2)}{n(n+b-3)}\times\left[(B\times Y)\text{S.S.}\right]$$

and $$\frac{(n-4)(n-2)[(A\times B\times Y)\text{S.S.}]}{n[n+(a-1)(b-1)-2]}$$

which are distributed as χ^2's with

$$\frac{(p-1)(a-1)(n-4)}{(n+a-3)},\ \frac{(p-1)(b-1)(n-4)}{(n+b-3)}$$

and $$\frac{(p-1)(a-1)(b-1)(n-4)}{[n+(a-1)(b-1)-2]}\text{ degrees}$$

of freedom respectively where n=degrees of freedom for error in the individual experiments. In this way if we consider the two-way tables of $(B\times C)$ and $(A\times C)$ we can work out similar χ^2-values and their d.f. for different interactions such as ($C\times$Years), $(A\times C\times Y)$ and $(B\times C\times Y)$.

For testing the significance of main effects and two factor interactions we proceed as follows :

(*i*) If χ^2-tests for $A\times$Years, $B\times$Years, $C\times$Years and $A\times B\times$Years, $A\times C\times Y$, $B\times C\times$Years are all significant, we may pool their respective unweighted sums of squares, if they are not heterogeneous and test the significance of main effects of A, B, C and interactions $A\times B$, $A\times C$ and $B\times C$ against the pooled mean sum of squares.

(*ii*) If some of the above components of interactions with years are heterogeneous but the χ^2 corresponding to each component is significant then pool those of the interaction components which are homogeneous and use the pooled mean sum of squares as the denominator for testing respective main effects and interactions. For example, if interaction $A \times B \times$ Years is significantly different from the remaining interaction components but all the interaction components are present then we will test the mean square for $A \times B$ interaction against the mean square for $A \times B \times$ Years. All other main effects and interactions will be tested against the pooled mean sum of remaining interactions (with years.).

(*iii*) If the χ^2-tests corresponding to some interaction components (say $A \times B \times$ Years) are not significant while all others are significant, the tests of significance of all main effects and two-factors interactions except those which are not significant (for example, $A \times B$) follow as in (*ii*) above. For these interactions which are absent the d.f. may be partitioned into suitable orthogonal contrasts and test each of these contrasts with their respective interactions with years.

4. Example

To illustrate the methods given in the foregoing sections we take an example of 3^3 confounded factorial experiment conducted at Dry Farming Research Station, Vallabhipur, (Gujarat) on Jowar crop during 1961-1962 to 1964-65. The treatments consisted of all combinations of 3 spacings between rows (designated as S_1, S_2 and S_3), 3 seed rates (designated as R_1, R_2 and R_3), and three manurial treatments (designated as F_0, F_1 and F_2). The experiment was laid out in 9 plot blocks replicated twice.

For combining the results over the four years the error variances have been tested by Bartlett's χ^2-test and were found to be heterogeneous. So, we have to see whether *Treatments* $\times$ *Years*' interaction is present. Since we have two-way tables of means, we can find out S.S. due to $(S \times Y)$, $(R \times Y)$, $(F \times Y)$, $(S \times R \times Y)$ and $(R \times F \times Y)$ interactions by weighted analysis leaving the highest order interaction $(S \times F \times R \times Y)$ which cannot be worked out from the available table of means. We will take up factors in pairs. Let us first take the factors S and R. As shown in Section 3, we first put the mean values for all the combinations of S and R for four years and work out the total weighted sum of squares for these nine treatment combinations. The mean values are given in Table 6 and the other steps also indicated therein;

TABLE 6

Treatment combinations of R and S	*Years*				$T_{(jk)}=\sum_i w_i x_{i(jk)}$
	1961	*1962*	*1963*	*1964*	
R_1S_1	464	285	919	554	1·024933
R_2S_1	476	343	855	295	1·002150
R_3S_1	377	275	743	213	0·802370
R_1S_2	466	444	1036	518	1 071562
R_2S_2	447	277	914	328	0·960053
R_3S_2	377	353	939	297	0·863712
R_1S_3	459	546	1003	449	1·062619
R_2S_3	415	298	1024	334	0·933658
R_3S_3	362	347	919	297	0·835684
$w_i=\frac{3r}{s_i^2}$	·001566	·000183	·000172	·000159	$\sum_i w_i=W=0\ 002080$
$\sum_{j,k} x_{i(jk)}=P_i$	3843	3168	8352	3285	
$\sum_{j,k} x_i^2{}_{(jk)}=X_i$	1657045	1170302	7817194	1304793	
w_iP_i	6·018130	·579744	1·436544	·522315	$\sum_{j,k} T_{(jk)}=\sum w_iP_i =8·556741=G$

$$C=\frac{G^2}{tW}=3911·208350$$

$$\text{Total S.S.} \quad =\sum_i w_iX_i-C=451·738841 \qquad \ldots(1)$$

$$\text{Years S.S.} \quad =\tfrac{1}{9}\sum_i w_iP_i^2-C=386·364271 \qquad \ldots(2)$$

$$\text{Treatment S.S.}=\sum_{(j,k)}\frac{T_{jk}^2}{W}-C=37·724020 \qquad \ldots(3)$$

Therefore, $(RS)\times$Years S.S. $=27·650550$, ...(4)

For obtaining the sums of squares due to ($R\times$Years) interaction, we take the marginal means of R under each year and conduct the weighted analysis as given in Table 7.

TABLE 7

Treatment	*Years*				$T_{j\cdot}=\sum_i x'_{ij}\cdot w_i'$
	1961	1962	1963	1964	
R_1	463	425	986	507	3.159114
R_2	446	306	931	319	2.895861
R_3	372	325	867	269	2.501766
$\frac{9r}{s_1^2}=w_i'$	0.004698	.000549	.000516	.000477	$\sum_i w_i'=W'=0.006240$
$\sum_j x'_{ij}=P'_i$	1281	1056	2784	1095	
$\sum_j x^2_{ij\cdot}=X_i'$	551669	379886	2590646	431171	
$w_i'P_i'$	6.018138	.579744	1.436544	.522315	$\sum_i w_i'P'_i=8\ 556741=G'$

Total S.S. $=431.531929$ ...(5)

S.S. for $R=35.0843$ ...(6)

Years S.S. $=386.36427$

Therefore, $R\times$Years$=10.083359$ ...(7)

Next we consider the marginal means of S under different years and work out the sums of squares for $S\times$Years interaction by weighted analysis following the same steps as already given above.

This gives us S.S. for $S\times Y=0.446007$ ...(8)

From these three values *viz.* $(RS)\times Y$ S.S.$=27.650550$

$R\times Y$ S.S.$=10.083359$.

$S\times Y$ S.S.$=\ 0.446007$.

From (4), (7) and (8), we get the S.S. due to $R\times S\times$Years interaction$=17.121184$, Similarly, by taking the other two tables of means *viz.* $(S\times F)$ and $(R\times F)$ and following the above steps we can work out the S.S. due to the remaining interactions *viz.* $(R\times Y)$, $(R\times F\times Y)$ and $(S\times F\times Y)$. The S.S. due to all these interactions are multiplied by suitable multiplying factors and χ^2-values have been obtained along with their degrees of freedom as given in table 8.

TABLE 8

(Effect × Years) Interaction	χ^2*—value*	*d.f.*	*significance*
$R\times Y$	10.083359	4.9	N.S.
$F\times Y$	22.715500	4.9	**
$S\times Y$	0.446007	4.9	N.S.
$R\times F\times Y$	13.597986	4.0	**
$R\times S\times Y$	17.121184	4.0	**
$F\times S\times Y$	21.359527	4.0	**

It is observed from the above table that $(F\times Y)$, $(R\times F\times Y)$, $(R\times S\times Y)$ and $(F\times S\times Y)$ interactions are present. $(R\times Y)$ interaction χ^2-value has just missed significance (Table value of χ^2 for 4.9 d.f. being 10.912). For all practical purposes we may take $(R\times Y)$ interaction also to be present. So the effects R, F, $R\times F$, $R\times S$ and $F\times S$ could be tested by F—test taking the respective interactions with years as the error. The (unweighted) S.S. due to various sources are given in the analysis of variance table 9 below :

TABLE 9

Analysis of variance

Source	*d.f.*	*S.S.*	*m.s.*
R	2	118126 5	59063.250
S	2	21871.5	10935.750
F	2	295039 5	147519.750
$R\times F$	4	27600.0	6900.000
$R\times S$	4	8596.0	2149.000
$F\times S$	4	9842.5	2460.625
$R\times Y$	6	36211.5	6035.250
$F\times Y$	6	171388.5	28564.750
$S\times Y$	6	30640.5	5106.750
$R\times F\times Y$	12	31618.0	2634.833
$R\times S\times Y$	12	38110.0	3175.833
$F\times S\times Y$	12	13358 0	1113.167

Among the interactions that are present, $(R\times Y)$, $(R\times F\times Y)$ and $(R\times S\times Y)$ are found to be homogeneous and hence the S.S. due to these interactions are pooled to test the effects of R, $(R\times F)$ and $(R\times S)$. The combined analysis of variance is given in table 10. Main effect of F is, tested against $(F\times Y)$ interaction and $(F\times S)$ interaction is tested against $(F\times S\times Y)$ interaction.

TABLE 10.

Analysis of variance

Source	*d f.*	*m.s.*	*F*
R	2	59063.25	16.7256**
$R\times F$	4	6900.0	1.953
$R\times S$	4	2149.0	—
$(R\times Y)+$ $(R\times F\times Y)+$ $(R\times S\times Y)$	30	3531.3	
F	2	147519.75	5.16*
$F\times Y$	6	28564.75	
$F\times S$	4	2460.625	2.21
$F\times S\times Y$	12	1113.167	

Since $(S\times Y)$ interaction is not present the effect of S can only be tested by dividing the 2 d.f. due to S into two orthogonal contrasts and testing them by their respective interactions with years as given in Cochran and Cox (1).

SUMMARY

In combining the results of similar experiments involving two or more factors under treatments there are several problems involved. Some of the interactions of treatment effects with years may be present and some of them may be absent. Further among the interactions that are present all of them may not be homogeneous. This makes the testing of treatment effects difficult.

The methods of combining results of such experiments have been discussed particularly when the results of individual experiments are available in the form of two-way tables of means. The methods have been illustrated with the help of an example of *a* 3^3 confounded factorial experiment conducted for 4 years in Gujarat State.

REFERENCES

1. Cochran, W.G. and Cox, G.M. : "Experimental Designs", 2nd edition, 1963. Analysis of the results of a series of Experiments. 545-567.

2. Cochran, W.G. : Problems arising in the analysis of groups of experiments. Journal of *Roy.* Stat. Soc. Suppl. 4, 102-118, 1937.

3. Yates, F., and Cochran, W.G. : The analysis of groups of experiments. Jour. Agri. Sci , 28, 556-580, 1938.